Crack-Growth Phenomena in Fatigued Poly(methylmethacrylate)

N. E. WATERS
School of Dental Surgery, Royal Dental Hospital, London, WC2, UK

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The conditions under which crack-growth occurs from a natural crack induced in one side of a tensile specimen of poly(methylmethacrylate), when subject to a cyclic tensile load, at 20 $\pm$ 3° C and 166 c/min, have been examined. The fracture surface can show three distinct regions, viz. a fissured "rough growth" area, a "smooth growth" area in which crack profiles extend across the width of the specimen, and a final area where catastrophic rupture has occurred. Under the conditions used, the fatigue life is mainly determined by the rate of growth of the crack through the rough-growth region. The number of cycles to failure has been determined for various values of the maximum bulk stress applied per cycle and of the initial crack length. The results appear to be consistent with a relationship put forward by Thomas, based on a tearing energy concept, which successfully predicts the dynamic cut-growth behaviour of certain natural and synthetic (SBR) gum rubbers. The conditions for the onset of catastrophic failure appear to be in accord with the Griffith criterion for brittle fracture.

1. Introduction

The concept of the energy balance theory of crack-growth, originally put forward by Griffith [1] for glass, has been applied successfully by a number of authors to a variety of materials. In particular, a generalisation of the Griffith theory employed by Rivlin and Thomas [2] has been used as a basis for unifying the mechanics of crack-growth in non-crystallising rubbers [3], glass [4], and brittle plastics [5], under static loading conditions; and also to formulate the behaviour of a crack in certain natural [6] and synthetic rubbers [7] subject to repeated or cyclic loading. In the latter case, incremental crack-growth occurs under stress conditions such that the maximum stress at the tip of the crack is insufficient to cause catastrophic rupture: an unstable condition which ensues when the supply of mechanical energy to the crack tip increases as the crack grows.

As will be explained subsequently, this incremental crack-growth has been shown to occur in a rigid plastic (polymethylmethacrylate) under similar conditions, the maximum bulk stress applied per cycle being low in comparison with the proportional limit of the material.

A considerable simplification in the analysis follows, since time-dependent relaxation effects due to the visco-elastic nature of the material may, at these stress levels, be neglected in comparison with the elastic strain energy, which is directly calculable from classical elasticity theory.

The success obtained in applying the above concepts to the fatigue fracture of poly(methylmethacrylate) suggests that this approach, although in part empirical, may be of value in systematising and collating the available fatigue data on a wider range of materials than has hitherto been attempted.

2. The Tearing Energy Concept

2.1. Static Crack-Growth

The energy balance theory of brittle, tensile fracture, originally put forward by Griffith [1] for glass, relates the amount of energy (γ) required to produce unit area of fracture surface, developed from an inherent flaw or cut in the material, to the resulting change in the elastic strain energy of the system. For a flaw size c, the ultimate tensile strength was shown to be given by the expression:

$$\sigma_{max} = K(E\gamma/c)^{\frac{1}{2}} \quad (1)$$

where E is the Young's modulus of the material and K is a dimensionless constant.

A generalisation of this criterion, which can be applied to any mode of deformation amenable to analysis, was given by Rivlin and Thomas [2] in the form

$$-\left.\frac{\partial W}{\partial A}\right|_l = T_c \quad (2)$$

Here, W is the total strain energy of the body; A, the interfacial area of the crack; and the suffix $_l$ indicates that the constrained boundaries of the body remain fixed during the growth of the crack such that no external work is done. The parameter T_c is the energy required to produce unit crack area as opposed to unit area of crack surface; hence $T_c = 2\gamma$. The specific fracture energy T_c is a characteristic of the material and is, in general, a function of the rate of crack propagation and temperature.

For a tensile strip having a cut or crack of length c in one edge, as illustrated schematically in fig. 1, it has been shown [2] that the initial stored energy of the test-piece resulting from the application of a constant tensile stress decreases from a value W_o (for $c = 0$) as the crack length increases according to the relation:

$$W = W_o - KW_o^1c^2t \quad (3)$$

Here, W_o^1 is the strain energy per unit volume in the bulk of the test-piece (i.e. remote from the crack), t is the thickness of the strip, and K is a function of the strain for elastomeric materials [8], which for brittle materials is equal to π on classical elasticity theory providing $c \geqslant t$.

On differentiating equation 3 with respect to c, it follows that

$$T = -\left.\frac{\partial W}{\partial A}\right|_l = -\left.\frac{1}{t}\frac{\partial W}{\partial c}\right|_l = 2\pi W_o^1 c \quad (4)$$

If it is now assumed that for unplasticised poly(methylmethacrylate), provided the maximum bulk stress is below the proportional limit, the energy loss due to time-dependent dissipative processes is negligible in comparison with the elastically-stored strain energy of the material, then $W_o^1 = \frac{1}{2}\sigma\epsilon = \sigma^2/2E$. Substitution for this value of W_o^1 in equation 3 gives

$$T = \frac{\pi c\,\sigma^2}{E} \quad (5)$$

which, after re-arrangement, is seen to be the well-known Griffith relationship given by equation 1. Since the energy available for tearing increases as the crack grows, equation 5 predicts that the fracture process under tensile conditions is unstable; thus, once crack-growth occurs, catastrophic rupture should ultimately ensue.

Clearly, if, for any mode of deformation, the change in the strain energy of the system is known as a function of the crack length c (analogous to equation 3 for the case of a tensile strip), then the Rivlin and Thomas generalisation of the Griffith criterion enables the tearing or fracture process under any conditions to be expressed in terms of a single parameter, the tearing energy T_c.

2.2. Dynamic Crack-Growth

Although equation 1 has been shown to predict the onset of catastrophic rupture for a variety of materials, there is some doubt as to whether it completely describes crack-growth phenomena. Thomas [6], for example, has shown that with natural rubber (NR), a strain-crystallising elastomer, whereas catastrophic rupture occurs for a fairly well-defined value of the specific rupture energy T_c, limited growth occurs for $T < T_c$, according to the approximate relation

$$\delta^{\frac{1}{2}} = kT \quad (6)$$

where σ is the increase in crack length and k is a constant. This "small-scale" crack-growth of NR has also been studied by Andrews [9], who has suggested that this phenomenon may be accounted for both qualitatively and quantitatively by postulating that the stress distribution around the crack remains stationary despite the advance of the crack tip. If the specimen is deformed in a cyclical manner to a given T value (i.e. to a given value of $-\partial W/\partial A$), it has been observed that incremental growth occurs which can be empirically characterised by a similar relation to that formulated for static growth, viz.

$$T^2 = G_d\left(\frac{dc}{dn}\right) \quad (7)$$

Here, dc/dn is the increase in the crack of length c in cm/c, and G_d is a constant referred to as the dynamic cut-growth constant. For NR, G_d is of the order of 2×10^{17} in cgs units [6].

The static and dynamic cut-growth of SBR gum rubbers has also been examined [7]; in this case, the results for both types of growth are consistent with the expression.

$$T^4 = G^1 \frac{dc}{dn} \quad (8)$$

At low frequencies of deformation, the crack-growth has a "static" as well as a "dynamic" component; at higher frequencies, however, the former disappears.

The success of the Thomas empirical equation in predicting the cut-growth behaviour of two dissimilar elastomers suggests that it may be equally applicable to cut-growth phenomena in a homogeneous, rigid polymer like poly(methylmethacrylate) at room temperature, but with different constants. If a general form of this equation is assumed, i.e.

$$G \frac{dc}{dn} = T^p \quad (9)$$

(where G and p are constants for any given material) for the growth of a crack in a tensile test-piece, substitution for T from equation 5 gives

$$G \int_{c_0}^{c} \frac{dc}{c^p} = \int_0^N \left(\frac{\pi\sigma^2}{E}\right)^p dn$$

i.e.

$$\frac{G}{(1-p)} \left[c^{1-p}\right]_{c_0}^{c} = \left(\frac{\pi\sigma^2}{E}\right)^p N \quad (10)$$

Here, c_0 is the initial length of the crack, and N the number of cycles for the crack to grow to a length c. In the majority of cases of practical interest $c \gg c_0$, and, as will be shown subsequently, $p \simeq 5$ for the sample of poly(methylmethacrylate) used; it is therefore possible to simplify (10) to give

$$\log N = -(p-1)\log c_0 - p \log \frac{\pi\sigma^2}{E} - \log \frac{(p-1)}{G} \quad (10a)$$

This equation may be used to determine p if the number of cycles to failure is determined for various values of c_0 at constant bulk stress σ. It is also possible to rearrange equation 10a to give

$$\log N - \log c_0 = -p \log \frac{\sigma^2 c_0}{E} - p \log \pi - \log \frac{(p-1)}{G} \quad (10b)$$

3. Experimental

3.1. Static Tear

Although measurements of the specific fracture energy of samples of poly(methylmethacrylate) have been made, e.g. Benbow and Roesler [5], it was considered desirable to check the published values in case any changes had been made to the commercially available material during the interim period. All specimens used in this study were cut from sheets of "Perspex" (Imperial Chemical Industries Ltd) having a nominal thickness of 0.79 mm ($\frac{1}{32}$ in.). For the static tear tests, parallel-sided strips 152.4 mm long by 12.7 $\pm$ 0.13 mm wide (6.0 in. long by 0.5 $\pm$ 0.005 in. wide), and of as near uniform thickness as possible, were selected. Cracks of varying lengths were made from one edge in the centre portion of each specimen.

Since it was apparent from other published work on crack propagation that the precise form of the crack tip was of paramount importance, methods of producing small, natural cracks were examined in some detail before the final method was evolved. This was as follows: a fret-saw cut, approximately 0.5 mm (0.02 in.) shorter than the desired crack length, was made with a blade 0.279 mm (0.011 in.) thick, as square as possible to the edge of the specimen. A new, single-sided razor blade was pressed into the notch thus formed until a natural crack was produced.

These cracks were then examined with a low-power microscope, and any specimens with crack profiles which were not apparently straight and square to the surfaces were rejected.

In some cases, the area around the tip of the crack showed the presence of residual strain when viewed under polarised light; all specimens were consequently annealed for 30 min at 100 $\pm$ 1° C and allowed to cool slowly to room temperature.

Tensile tests on these specimens were carried out at 20 $\pm$ 2° C on a Hounsfield Tensometer at a crosshead speed of 1.58 mm/min ($\frac{1}{16}$ in./min), using "Quick-grip" chucks with a free length of 76.2 mm (3.0 in.).

3.2. Dynamic Fatigue

The apparatus used for the dynamic crack-growth tests, shown diagrammatically in fig. 1, was adapted from a 2-ton die-stamping press. The specimen, S, was suspended vertically from a sliding head, A, which reciprocated to and fro with a throw of 25.4 mm (1.0 in.) at 166 rev/min, between grooved tracks, being actuated by eccentric cam, B, driven by a $\frac{1}{2}$ hp, 3-phase electric motor. The lower specimen clamp was fixed to the upper end of a helical spring, H, the lower end of which was firmly anchored.

Apart from an initial pre-tension of 4.5 kg (10 lb), the spring characteristic, as determined on a Hounsfield Tensometer, was linear and had a value of 859 kg/m (48 lb/in.) extension. The number of cycles to failure was recorded on a trigger-actuated counter, provided with a cut-out which operated when the specimen fractured.

Dumb-bell-shaped specimens 152.4 mm (6 in.) long were cut from 0.79 mm ($\frac{1}{32}$ in.) sheet "Perspex" with the aid of metal templates. The width of the central portion was varied between 15.2 and 40.6 mm (0.6 and 1.6 in.), so that the bulk stresses applied could be varied between 60 and 160 kg/cm^2 (800 and 2000 lb/in.^{2}). Cracks of various lengths were produced in one edge of the central parallel-sided portion of each test-piece, and all specimens were annealed by the procedures outlined in section 3.1 above.

4. Results and Discussion

4.1. Fracture Morphography and Rate of Crack-Growth

It has not been found possible, during the course of a fatigue experiment, to record the variation of crack length c with number of cycles by direct measurement. Invariably, the initial growth of the crack from the incision is not uniform over the whole crack-front, but progresses sporadically in different regions of the front in a series of small tongues. Examination of the fracture surface after catastrophic rupture shows that, in some cases, regular growth of the whole crack-front is apparent in an area immediately in front of the site from which catastrophic rupture has occurred. All measurements of dc/dn as a function of c have been made in this area.

The general correlation between surface morphology and the rate of crack-growth is shown diagrammatically in fig. 2a. The irregular or rough-growth region adjacent to the incision is characterised by a number of fissures fanning out in the direction of crack propagation. As noted above, this surface roughness can, in some cases, disappear; the crack then enters a "smooth growth" region, where successive, stationary, crack profiles may be seen extending across the whole width of the test-piece. The incremental growth at this stage gradually increases until, ultimately, catastrophic fracture occurs, and secondary crack frontiers with foci at both surface and internal imperfections are

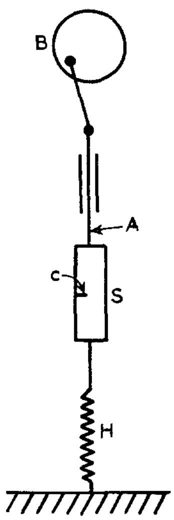


Figure 1 Diagrammatic representation of apparatus used for cycling loading.

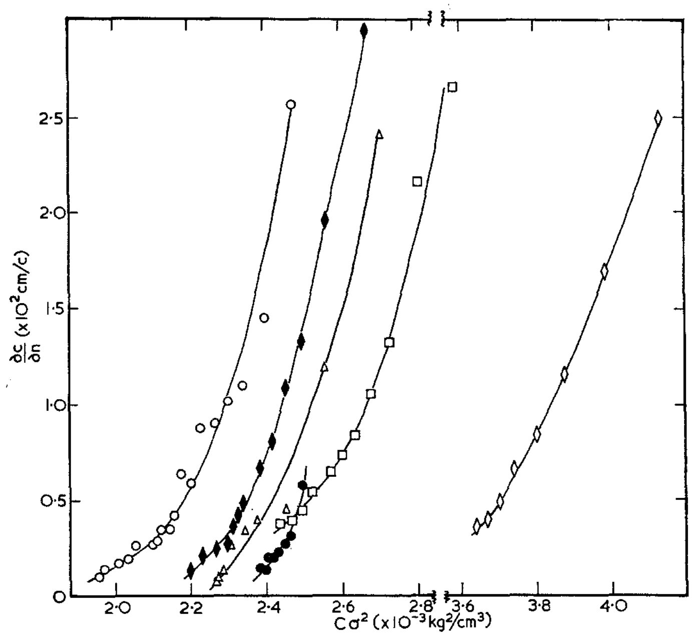


Figure 3 Variation of rate of crack-growth per cycle with $c\sigma^2$ for smooth growth.

usually visible. Figs. 2b, c, and d show these effects in more detail. When viewed microscopically by reflected white light, the crack surfaces are coloured, varying from a pink-orange to a green-blue shade. These colour effects have been described in the literature by Berry [10] and others, and are attributed to an interference effect in a thin layer of structurally modified (or oriented) material at the fracture plane. On matching the two fracture surfaces, the colouring is found to be roughly complementary; a pinkish area on one surface is matched by a greenish area on its fellow. A certain periodicity of colour is also apparent between the profiles in the smooth-growth region.

4.2. Smooth-Growth Region

Since the variation of dc/dn with T could be obtained in the smooth-growth region, by measuring the positions of the crack profiles with a travelling microscope, in the first instance an attempt was made to apply equation 9 directly, with p and G as unknowns. As may be seen from fig. 3, the curves are reasonably regular although dispersed along the abscissa which is proportional to T. Since superposition of these curves could roughly be obtained by translation along the T axis, the possibility that this dispersion might be due to slight differences in the modulus of the specimens was examined. Check measurements of the modulus on portions of the broken specimens, however, failed to establish a difference. Again, no correlation was found between the T_c value and the maximum bulk stress applied per cycle, or number of cycles to failure. No obvious explanation can therefore be put forward to account for the spread in these results. A double logarithmic plot gives a mean value of p, from the slopes of the best straight lines, of approximately 15, which cannot pos-

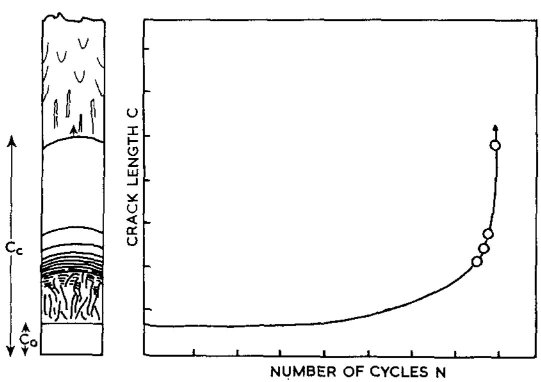
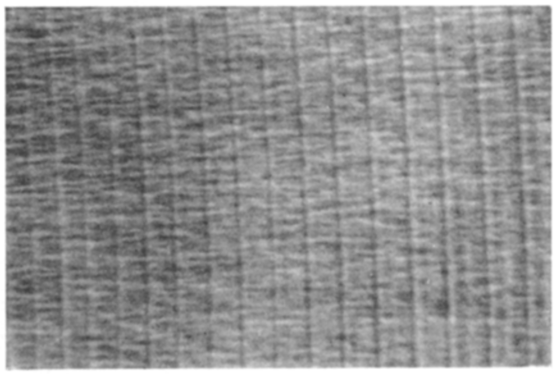
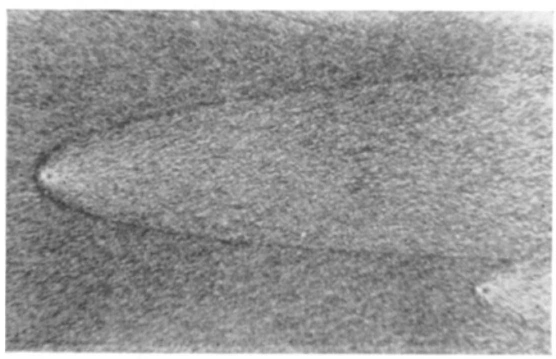


Figure 2a Diagrammatic correlation between rate of crack-growth dc/dn and surface morphology in the fracture plane.

(b)



(c)



(d)

Figure 2 (b) Transition from rough to smooth growth. (c) Crack profiles in smooth-growth region. (d) Secondary crack frontier in catastrophic fracture surface. (Magnification × 130. Direction of crack-growth is from left to right.)

sibly account for the results in the rough-growth region. Since dc/dn cannot be determined with any degree of accuracy in this latter region, the integrated form of equation 8 must be applied.

4.3. Variation of the Fatigue-Life N with Initial Crack Length c_0 for Constant Values of the Maximum Applied Bulk Stress σ

The variation of the fatigue-life N (cycles) with the initial crack length c_0 for five values of the maximum applied stress between 60 and 120 kg/cm² is shown in fig. 4. A figure of approximately 800 kg/cm² for the tensile strength of "Perspex" is given by the manufacturers. The applied, maximum bulk stresses were consequently well below the proportional limit (see derivation of equation 5), assuming relaxation effects are small enough to be neglected.

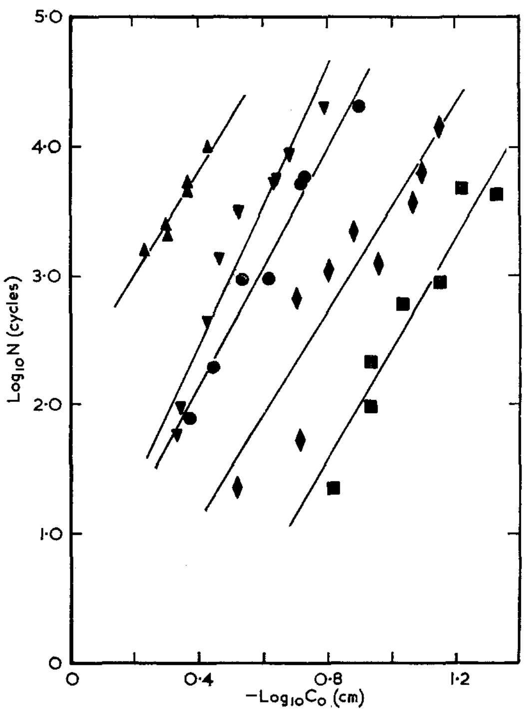


Figure 4 Dependence of number of cycles to failure (N) on initial flaw size (c_0) for various maximum bulk stresses σ (kg/cm²): ▲ 63.7; ▼ 71.6; ● 87.1; ◆ 119.1; ■ 158.8.

Reasonably linear plots are seen to be obtained for all five stress values, in accordance with equation 10a. A statistical analysis of these results showed that the slopes of the regression lines were not statistically different. The mean value of the parameter p calculated from these slopes is 5.54.

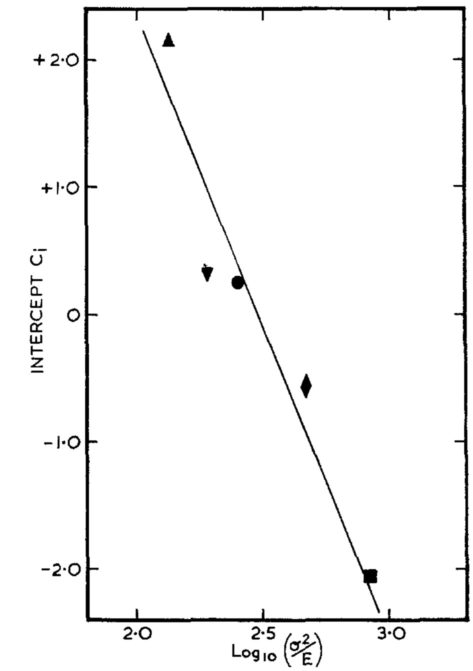


Figure 5 Intercepts from $\log_{10} N$ versus $-\log^{10} c$ of fig. 4 plotted against $\log_{10} \sigma^2/E$.

The consistency of the data with regard to the dependence on σ may be tested in two ways. Fig. 5 shows the values of the intercepts on the ordinate of fig. 4 plotted against log (σ^2/E), the slope of which, from equation 10a, should have the value $-p$. The line drawn in fig. 5 has a slope of -5.5; the agreement may be seen to be satisfactory.

An alternative method of checking the consistency of the data with respect to all three variables is shown in fig. 6. This graph incorporates the results of a number of specimens for which the maximum bulk stress differed from the five constant values noted above. Here, both the ordinate and abscissa are functions of c_0, which is not entirely satisfactory, nevertheless the superposition of results, which shows no systematic variation with σ, lends support to the general applicability of the Thomas empirical relation (equation 9) to the data obtained. Furthermore, the slope of the best straight line, fitted on a least-squares basis, gives a value of

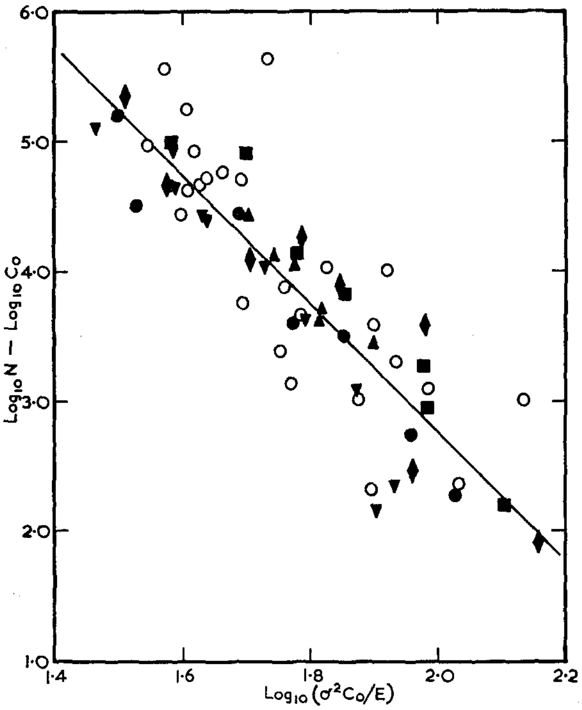


Figure 6 Data of fig. 4 replotted according to equation 10b, together with other results (open circles) obtained at intermediate levels of the maximum bulk stress applied per cycle (σ).

$p = 4.9$, in good agreement with the mean value of 5.5 obtained from the data of fig. 4.

From the intercept of fig. 6, the crack-growth constant G obtained using equation 10b was 3.6×10^9 in cgs units.

4.4. Comparison of T_c Values obtained from Static and Dynamic Tests

Assuming the Griffith criterion (equation 2) to hold, equation 5 predicts that a plot of c versus $1/\sigma^2$ should yield a straight line passing through the origin with a slope of ET/π. Despite some experimental scatter, the results obtained (see fig. 7) appear to be in accord with this relationship. By applying the method of least squares to these results, the slope with its standard error was found to be $0.529 \pm 0.078 \times 10^{10}$ gm²/cm³. Using a value of Young's modulus E of $2.97 \pm 0.026 \times 10^{10}$ dynes/cm² (nine observations) as determined on specimens mounted as a cantilever beam, the computed value of $T = 5.385 \pm 0.792 \times 10^5$ ergs/cm², giving a fracture surface energy of $2.693 \pm 0.396 \times 10^5$ ergs/cm².

Since all specimens subjected to cyclic loading ultimately failed by catastrophic rupture, an

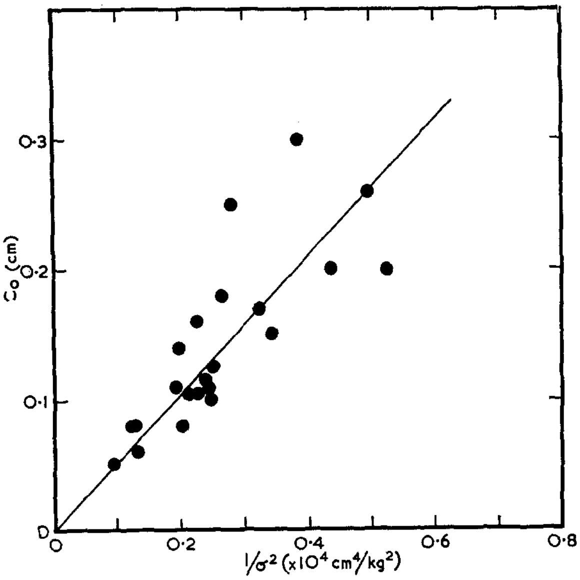


Figure 7 Variation of tensile strength σ with depth of natural crack c_0 (static test).

estimate of T_c, the critical tearing energy, could also be obtained in these cases by recording the crack-length c_c of the last growth profile. These results are shown in fig. 8, the symbols used denoting the conditions under which fracture occurred (see also fig. 4). Again a linear plot is seen to obtain. The value of γ from the best straight line through the origin together with its standard error is $2.311 \pm 0.054 \times 10^5$ ergs/cm².

These values of γ are in reasonable agreement with other published figures. Using a cleavage technique, Benbow and Roesler [5] give a value of 4.9×10^5 ergs/cm² for a sample of poly(methylmethacrylate) from the same source.

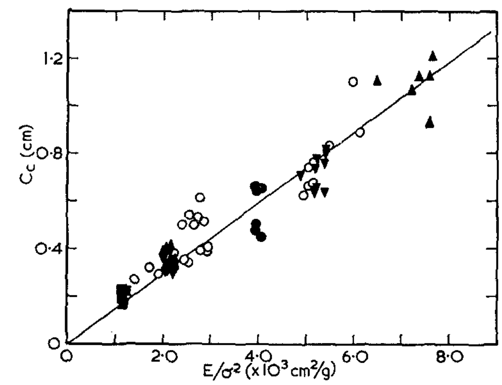


Figure 8 Variation of maximum bulk stress per cycle σ with crack length from which catastrophic rupture occurred (c_c).

Berry [11], on material manufactured by Röhm and Haas, quotes values of 1.40 ± 0.07 and $2.1 \pm 0.5 \times 10^5$ ergs/cm², for a cleavage method, and for direct tension, respectively. In order to test the statistical significance of the difference between the values of γ determined from the static and dynamic results, the statistic $\frac{|b_1 - b_2|}{\sqrt{(\sigma_1^2 + \sigma_2^2)}}$ was computed, the value found being 2.6. In this expression $|b_1 - b_2|$ is the difference in the numerical values for the two slopes and $\sqrt{(\sigma_1^2 + \sigma_2^2)}$ is the standard error of $|b_1 - b_2|$, where σ_1^2 is the variance of b_1, and σ_2^2 that of b_2. A significant difference is considered to be established if this statistic is greater than 2 on the normal distribution. The γ values are therefore just significantly different. A possible explanation for this difference is that, since the rate of application of the stress is lower in the tensile test, relaxation effects due to molecular orientation at the crack tip will be greater and result in an apparently larger value of the fracture surface energy.

5. Conclusions

In common with certain natural and synthetic rubbers, unplasticised poly(methylmethacrylate) at room temperature exhibits the phenomenon of small-scale crack-growth for values of the parameter T which are well below the critical tearing energy T_c of the material as deduced from the Griffith theory of brittle fracture.

The rate of crack-growth dc/dn has been found to depend on the surface roughness of the crack and, under the conditions of the present investigation, the fatigue-life is determined by the time spent in a "rough growth" region. Smooth growth, where it was observed, occurs immediately prior to catastrophic rupture.

It has been found that the results of fatigue measurements at 20° C in simple extension are in accord with an empirical expression put forward by Thomas [6] involving two constants: $G\,(dc/dn) = T^p$; $G = 3.65 \times 10^9$ in cgs units, $p \simeq 5$, dc/dn is the mean incremental growth per cycle, and T an energy parameter given by equation 2.

In these fatigue tests, failure ultimately occurred through catastrophic rupture of the specimens at a well-defined value of $T (= T_c)$ in accordance with the Griffith theory of brittle fracture. The value of the fracture surface energy $(= T_c/2)$ from these dynamic tests (2.3×10^5 ergs/cm²) was slightly lower than the value obtained from tensile tests.

How far the present results would be applicable to other commercially available poly(methylmethacrylate) is, at present, a matter for speculation, as is the effect of temperature and frequency of loading. It has been shown, for example, that the fracture surface energy of "Plexiglass" (Röhm and Haas Co) is dependent on the molecular weight [12] and temperature [13]. In addition, it has been observed with both polystyrene [5] and poly(methylmethacrylate) [12] that, under static growth conditions, the mode of propagation can change from "slow" to "fast" in an apparently arbitrary manner. The degree of perfection of the initial crack profile and the number and size of any internal flaws must clearly be of some importance in determining whether smooth or rough growth occurs.

Evaluation of the empirical constants p and G for other rigid polymers should help to clarify their physical significance, and thus to indicate the lines on which a general theory of fatigue fracture could be formulated.

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References

1. A. A. GRIFFITH, *Trans. Roy. Soc. London* **A221** (1920) 163; *Proc. Int. Congr. Appl. Mech.* (Delft) (1924) 55.
2. R. S. RIVLIN and A. G. THOMAS, *J. Polymer Sci.* **10** (1953) 291.
3. H. W. GREENSMITH and A. G. THOMAS, *ibid*, **18** (1955) 189.
4. F. C. ROESLER, *Proc. Phys. Soc.* (London) **B69** (1956) 981.
5. J. J. BENBOW and F. C. ROESLER, *ibid*, **B70** (1957) 201.
6. A. G. THOMAS, *J. Polymer Sci.* **31** (1958) 467.
7. G. J. LAKE and P. B. LINDLEY, *J. Appl. Polymer Sci.* **8** (1964) 707.
8. H. W. GREENSMITH, *ibid*, **7** (1963) 993.
9. E. H. ANDREWS, *J. Appl. Phys.* **32** (1961) 542.
10. J. P. BERRY, *Nature* **185** (1960) 91.
11. *Idem*, *J. Appl. Phys.* **34** (1963) 62.
12. *Idem*, *J. Polymer Sci.* **2A** (1964) 4069.
13. *Idem*, *ibid*, **1A** (1963) 993.